Trajectory tracking for underactuated mechanical systems using feedback regularization

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Abstract

Almost-global stabilization (AGS) about a set point for a trajectory of a fully actuated mechanical system (called simple mechanical system or SMS) evolving on a compact manifold has been studied in an early paper by Koditschek ([3]). AGS implies stability in the sense of Lyapunov for almost-all initial conditions in the phase space, which is the tangent bundle of the manifold. In the energy-like Lyapunov function employed to demonstrate AGS, the potential energy arises from a Morse function with a unique minimum - called a "navigation function" in the literature. Such functions are known to exist on compact manifolds. Recently, we have proposed a control law for almost-global tracking of a smooth reference trajectory for a fully actuated SMS on a compact manifold([6]). This is achieved by AGS of the error dynamics about the minimum of the navigation function lifted to the phase space. The error dynamics is an SMS on the Riemannian manifold that describes the evolution of an error trajectory which is chosen according to specified compatibility conditions.

Stabilization of underactuated systems has been studied using controlled Lagrangians in [1] and [2]. A natural question that arises after this is: what tracking objectives can be achieved for an underactuated SMS on a compact manifold? To address this question we first look at the problem of almost-global asymptotic tracking (AGAT) of only the unactuated configuration variables in an underactuated, interconnected SMS. This problem has been studied in [4] and [5] using a technique called "feedback regularization" for tracking of a hoop robot and spherical robot, both of which are internally actuated. Our main objective is to extend this framework to include a larger class of underactuated mechanical systems.

We study two problems which fall under this class: 1. attitude tracking of a rigid body evolving on $SO(3)$ and actuated by 3 internal rotors and 2. tracking of the underactuated joint in a 2 link robot (called the "pendubot"), evolving on $S^1 \times S^1$. Our approach can be broadly described in three steps. In the first step, the unactuated dynamics is isolated from the actuated part by expressing the actuated variables in terms of the control and the coupling terms. In the second step, the error dynamics for the unactuated part is formulated - this lacks a Riemannian struture. In the third step, a new control is introduced that annuls the terms which destroy the Riemannian structure of the unactuated error susbsytem. This is called feedback regularization. As the error dynamics is now a fully actuated SMS on the manifold where the unactuated variable evolves, the control law in [3] is applied for AGS. AGS of the error trajectory implies the AGAT of the reference trajectory. Under the control law proposed, the trajectory of the actuated part remains bounded. Simulation experiments on both the systems are presented to demonstrate the performance of the control law.

Keywords
SMS, feedback regularization, geometric mechanics, almost-global tracking, navigation function

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